# On the energy balance of a turbulent mixing layer

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An analysis of experimental results shows that the three most important terms in the energy balance of a mixing layer—the production, diffusion and dissipation terms—may be expressed in terms of an eddy viscosity, eddy diffusivity for turbulent energy, and a microscale, respectively, all being constant for a given cross-section. A first-order solution calculated for the resulting differential equation yields a turbulent intensity profile in qualitative agreement with experiment. The peak value of the turbulent intensity is found to depend on a dissipation constant and on the ratio of eddy diffusivity for turbulent energy to eddy viscosity. Experiments carried out with the intention of reducing turbulent intensities by means of extraneous vortices, generated by small delta wings, have shown slight increases in intensity, presumably because eddy viscosity increases with eddy diffusivity.

## 1. Introduction

The thin transition region which forms at a velocity discontinuity in a fluid is termed a 'mixing layer'. When a high-speed jet exhausts into stationary fluid, for example, a mixing layer develops on the jet boundaries. Sufficiently far from the exhaust, the mixing layers merge into a 'fully developed' jet, but before this stage is reached there is a practically important region in which the mixing layers grow unaffected by the existence of their opposites across the jet. It is now believed that much of the aerodynamic noise of jets originates from the mixing layer rather than from the fully developed jet (Lilley 1958; Ribner 1958). The amount of noise radiated is proportional to a high power of the turbulent intensity and how the intensity of turbulence in a mixing layer is maintained becomes a practically important question.

An early idea aiming at jet noise suppression was the generation of vortices in the mixing layer by serrations or teeth on the exhaust nozzle (Westley & Lilley 1952). Such devices undoubtedly do suppress noise, but it is still not known in exactly what manner they produce their effect. There are several possible explanations. For example, the increased diffusive capacity of turbulence due to the externally created vortices may lead to a reduction of the mean velocity gradient or of the turbulent intensity, or both. Either effect is known to reduce noise. Without going into further details on this point, it would be clearly important to know whether the externally applied agitation is in fact likely to reduce the turbulent intensity.

While there is no lack of theoretical investigations on the mean velocity profile in a mixing layer (e.g. Schlichting 1960 for a summary, and also Townsend 1956), little work seems to have been done on the distribution of turbulent intensity in such a layer. Townsend (1956) gives the self-preserving form of the energy equation and quotes the experimental results of Liepmann & Laufer (1947), but this is where matters stand. It is one purpose of the present paper to construct an approximate theory of energy balance in the mixing layer and thus relate the turbulent intensity to the diffusive, dissipative, etc., properties of the turbulence. Such a theory will then serve as a guide in assessing the probable effects of extraneous vortices on the mixing layer.

External vortices in a mixing layer may be created by arranging delta-wing shaped 'vortex generators' at the nozzle of an open-jet wind tunnel. Some experiments carried out in this manner are described below. Turbulent intensity profiles were measured by means of a commercially available constant-temperature hot-wire anemometer.

The more important results of the investigations are as follows. (*a*) The theory shows that the peak turbulent intensity depends on an energy dissipation constant and (although rather weakly) on the ratio of the exchange coefficients for turbulent energy and for mean flow momentum (a quantity akin to Prandtl number). A doubling of this latter ratio would result in a reduction of the peak turbulent intensity by about 20 %. (*b*) The experiments demonstrate that the extraneous vortices tend to increase the peak turbulent intensity slightly or at least keep it constant. A detailed consideration of the results suggests that the above mentioned ratio of the exchange coefficients remains more or less constant.

The results discussed here are in accord with the findings of Corcos (1959) on corrugated nozzles. Corcos also finds a peak turbulent intensity equal to or slightly higher than that measured in a simple, circular jet. The well-known noise-suppressing effect of these devices may therefore be tentatively ascribed to the great reduction they produce in the volume occupied by high-intensity turbulence accompanied by strong mean shear.

In the case of serrations or teeth on a nozzle the causes of noise reduction must be sought for in a reduction of the mean velocity gradient, because this is the one outstanding effect produced by the extraneous vortices. It is in fact well known that the teeth on a nozzle reduce mainly the low frequency 'shear noise' whereas the high frequency 'self-noise' of the turbulence (terminology of Lilley 1958) may even be increased by the introduction of such devices.

## 2. Notation

In general the notation follows Townsend (1956); thus:

$U, V$ are components of the mean velocity,
$u, v, w$ are components of the velocity fluctuation,
$q^2 = u^2+v^2+w^2$ is the turbulent intensity,
$\epsilon$ is the energy dissipation per unit mass,
$U_1$ is the jet exit velocity, directed along the $x$-co-ordinate,
$\eta = y/x$ is the non-dimensional cross-flow co-ordinate,
$x$ is distance along the jet axis, measured from the nozzle exit.

Other symbols of importance defined in the text are:

$\alpha$ shear coefficient,
$\Lambda$ dissipation constant,
$\gamma$ diffusion constant,
$\nu_T$ eddy viscosity,
$R_T = U_1 x/\nu_T$ eddy Reynolds number,
$f(\eta)$ non-dimensional mean velocity profile,
$g(\eta)$ non-dimensional intensity profile.

## 3. The basic equations

The kinetic energy of the velocity fluctuations in a turbulent mixing layer is subject to the equation

$$\left(U\frac{\partial \frac{1}{2}\overline{q^2}}{\partial x} + V\frac{\partial \frac{1}{2}\overline{q^2}}{\partial y}\right) + \overline{uv}\frac{\partial U}{\partial y} = -\frac{\partial}{\partial y}(\tfrac{1}{2}\overline{q^2 v} + \overline{pv}) - \epsilon. \qquad (1)$$

The various terms in this equation describe respectively the advection of turbulent energy by the mean flow, the production of turbulent energy by an interaction of the mean velocity gradient and shear stress, diffusion and dissipation of turbulent energy. The equation is accurate to the usual boundary-layer approximation. Townsend (1956) gives a similar equation with one extra term expressing the production of turbulent energy by normal stresses. Direct measurements made by Liepman & Laufer (1947) show that excepting the outermost region of the mixing layer this extra term is negligible compared to the others in equation (1). Since, in the following considerations attention is focused mainly on phenomena near the centre of the mixing layer, the extra term has been neglected.

A companion relationship is the equation of mean motion. To the same degree of approximation as equation (1), neglecting the viscous term, this may be written

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + \frac{\partial \overline{uv}}{\partial y} = 0. \qquad (2)$$

The cross-flow component of the mean velocity, $V$, may be eliminated by making use of the equation of continuity,

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \qquad (3)$$

Also, measurements show that the flow is nearly self preserving beyond $R_x = 4\times 10^5$, so that one may assume the mean velocity and the turbulent energy distributions to be of the form

$$U = U_1 f(y/x) = U_1 f(\eta), \quad \overline{q^2} = U_1^2 g(y/x) = U_1^2 g(\eta). \qquad (4)$$

Similar self-preserving forms hold for the distribution of the energy dissipation $\epsilon$, the Reynolds stress $\overline{uv}$ and the diffusion term. However, the number of independent equations, after the elimination of $V$ by the aid of equation (3), is only two, so that if the problem is to be made determinate, means must be found for relating these other quantities to the two unknown functions defined in equation (4).

## 4. The diffusion term

An examination of the experimental results of Liepmann & Laufer, as presented by Townsend (1956, p 181), suggests that the diffusion term may be represented as a 'gradient diffusion',

$$\tfrac{1}{2}\overline{q^2v}+\overline{pv} = -D(\partial\overline{q^2}/\partial y). \tag{5}$$

Taking into account the self-preserving nature of the flow the diffusion coefficient may be expressed as

$$D = \gamma U_1 x, \tag{6}$$

with $\gamma$ a diffusion constant.

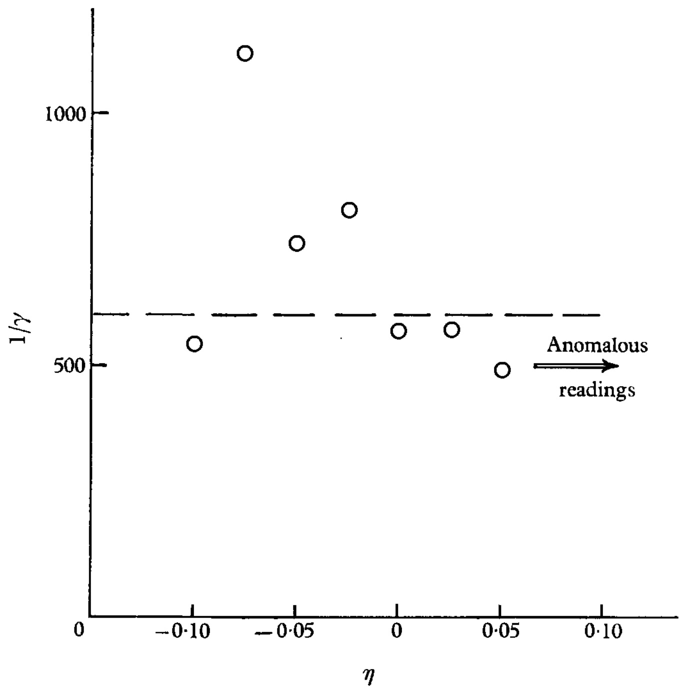


FIGURE 1. Diffusion constant $\gamma$ in a mixing layer.

Dividing the measured values of the diffusion term by the second derivative of the $\overline{q^2}$ profile, one may obtain the diffusion constant $\gamma$ at several values of the independent variable $\eta = y/x$:

$$\gamma = \frac{1}{U_1 x}\frac{\partial(\tfrac{1}{2}\overline{q^2v}+\overline{pv})/\partial y}{\partial^2\overline{q^2}/\partial y^2}. \tag{7}$$

A double differentiation of the measured $\overline{q^2}$ profile is involved and the accuracy of this is inherently poor. In order to obtain reasonably reliable results, a tenth-degree polynomial was fitted to the $\overline{q^2}$ profile measured by Liepmann & Laufer. The values of the diffusion term $\partial(\tfrac{1}{2}\overline{q^2v}+\overline{pv})/\partial y$ were obtained in the experiments of Liepmann & Laufer (1947) by difference between all the other production, dissipation, etc., terms so as to make equation (1) balance. The resulting distribution of this term seems to contain some important errors. The integral of the diffusion term over all $y$ should, for example, come to zero, since the integrand is a space derivative, vanishing at large enough absolute values of $y$. This is certainly not the case for the experimental curve (see Townsend 1956, figure 8·4). Particularly at relatively large positive values of $\eta = y/x$ the measured (or rather computed from other measurements) results appear to show too small a gain of turbulent energy by diffusion.

Thus the values of $\gamma$ calculated as indicated in equation (7) may be expected to show considerable scatter. Figure 1 shows the results of the calculations between $-0{\cdot}10 \leqslant \eta \leqslant 0{\cdot}05$. At higher positive values the results become nonsensical: $\gamma$ fluctuates rapidly and becomes negative in certain places. It is in this region that the whole experimental technique becomes unreliable, the turbulent velocity fluctuations being of the same order of magnitude as the mean velocity. Townsend (1956) has already voiced some doubts about the validity of measurements of the shear coefficient, $\overline{uv}/\overline{u^2}$, in this region.

Allowing for all the probable inaccuracies, the fact still remains that the curve representing the diffusion term is very much of the right shape to be proportional to $d\overline{q^2}/dy$. Thus the gradient diffusion hypothesis, $\gamma = \text{const.}$, will here be accepted as reasonable. From figure 1 one may estimate a crude approximate value for the diffusion constant,

$$\gamma \sim 1/600. \tag{8}$$

## 5. The dissipation term

In the observations of Liepmann & Laufer (1947) the energy dissipation $\epsilon$ was calculated from the well-known relation

$$\epsilon = 15\nu\overline{u^2}/\lambda^2, \tag{9}$$

where $\lambda$ is the microscale of turbulence and $\nu$ the kinematic viscosity of the fluid. An important experimental finding was that the microscale $\lambda$ is constant for any cross-section. Then, the self-preserving nature of the flow requires that

$$\frac{\lambda^2}{\nu} = \text{const.}\,\frac{x}{U_1}. \tag{10}$$

This relationship has been explicitly confirmed by Liepmann & Laufer for the self-preserving range. After substitution into equation (9) one finds the expression for energy dissipation

$$\epsilon = \Lambda\frac{U_1^3}{x}\frac{\overline{q^2}}{U_1^2} = \Lambda\frac{U_1^3}{x}g(\eta), \tag{11}$$

where $\Lambda$ is an energy dissipation constant, and where proportionality of $\overline{u^2}$ and $\overline{q^2}$ has been assumed. The single constant $\Lambda$ now characterizes the energy dissipation in a mixing layer in the same way as the constant $\gamma$ characterizes the diffusion of turbulent energy. To calculate $\Lambda$ we make use of the empirical relationships given by Liepmann & Laufer,

$$\lambda = 0{\cdot}04\sqrt{x}. \tag{12}$$

The assumption that $\overline{u^2}$ and $\overline{q^2}$ are proportional is not strictly true, but in the absence of accurate data on the third velocity component $\overline{w^2}$ a reasonable hypothesis is

$$\overline{q^2} = 2{\cdot}25\overline{u^2}. \tag{13}$$

Townsend (1956) assumes $\overline{q^2} = \frac{3}{2}(\overline{u^2}+\overline{v^2})$ and equation (13) is obtained from this by setting $\overline{u^2} = 2\overline{v^2}$, which describes the measured profiles near the centre of the mixing layer reasonably accurately. Observing now that $U_1$ in the experiments of Liepmann & Laufer was 18 metres per second the value of $\Lambda$ may be calculated. One finds approximately $\Lambda \sim \frac{1}{3}$.

## 6. The production term

The principle of 'structural similarity' (Townsend 1961) leads to the postulate

$$\overline{uv} = \alpha\overline{q^2} \tag{14}$$

with $\alpha$ = const. According to the experimental evidence, although the coefficient $\alpha$ varies more slowly than $\overline{q^2}$ itself, it would be not quite accurate to regard $\alpha$ as constant over a cross-section or even over the whole field. The main discrepancies occur, however, at the low-velocity edge of the flow and, as pointed out by Townsend (1956), may to some extent be of instrumental origin. There is also some supporting evidence from other kinds of shear flow (wakes, boundary layers, channels) to show that equation (14) is approximately valid over at least a substantial proportion of the flow cross-section. Thus it would not be unrealistic to accept this equation as a working hypothesis.

At the same time, it is also well known that the assumption of an eddy viscosity $\nu_T$ constant over a cross-section furnishes a reasonably accurate description of the mean velocity profile. On examining the details it turns out that the eddy viscosity, defined by the equation

$$\overline{uv} = -\nu_T\frac{\partial U}{\partial y}, \tag{15}$$

is indeed nearly constant between $-0{\cdot}1 \leqslant \eta \leqslant 0{\cdot}05$ (Townsend's co-ordinates), but decreases considerably near the low-velocity edge of the flow. Again, the directly measured values of $\overline{uv}$ in this region are subject to considerable instrumental error and it is not certain how far the observed variation of $\nu_T$ is real. A comparison of the measured mean velocity profile with calculated ones suggests that the eddy viscosity decreases somewhat near the edges of the flow. On the whole, the accuracy of equation (15) seems to be in the same category as that of equation (14): both represent realistic working hypotheses with little to choose between the two in regard to probable accuracy.

Expressed in terms of the functions $f(\eta)$ or $g(\eta)$, equations (14) and (15) become

$$\overline{uv} = \alpha U_1^2 g, \tag{14a}$$

$$\overline{uv} = -\frac{\nu_T U_1}{x}\frac{df}{d\eta}. \tag{15a}$$

In the course of calculations it becomes evident that equation (15$a$) is much more convenient to use than equation (14$a$).

## 7. The mean velocity profile

If the turbulent shear stress is expressed in terms of an eddy viscosity, as has been done in equation ($15a$), the equation of mean motion becomes independent of the distribution of turbulent intensity and may be solved directly. The solution is the well-known one due to Görtler (see Schlichting 1960) which is best expressed in terms of the integrated velocity distribution

$$F(\eta) = \int_{\eta_0}^{\eta} f(\eta')\,d\eta'. \tag{16}$$

Here the co-ordinate $\eta = \eta_0$ refers to the point where the cross-component $V$ of the mean velocity is zero; in Townsend's co-ordinates $\eta_0 = 0{\cdot}05$.

As may be seen already from equation ($15a$) the only numerical factor entering Görtler's solution is the Reynolds number of the turbulence,

$$R_T = U_1 x/\nu_T. \tag{17}$$

The value of this may be estimated directly, by means of equation (15), or by fitting Görtler's profile to the experimental data. With the exception of the low-velocity region there is fair agreement between the two determinations. On the basis of the measurements of Liepmann & Laufer (1947) and Reichardt (1942; see Schlichting 1960) one may estimate $R_T$ to be about 600.

## 8. The distribution of turbulent intensity

When the hypotheses contained in equations (5), (6), (12) and ($15a$) are introduced into the energy balance, equation (1), one finds after some minor reductions that

$$\frac{d^2g}{d\eta^2} - \frac{\Lambda}{\gamma} g = -\frac{1}{R_T\gamma} F''^2 - \frac{1}{2\gamma}\frac{dg}{d\eta} F. \tag{18}$$

The second term on the right of this equation is zero at the position of maximum turbulent intensity and also where $V = 0$ (cf. equation (16)). This term represents the advection of turbulent energy by the mean flow and has been calculated from direct measurements by Townsend (1956). As may be expected, its value is small near the centre of the mixing layer (where $dg/d\eta = 0$) and on the low-velocity side of the centre (where one finds the point $F = 0$), but it is a dominant term well away from the centre of the layer, on the high-velocity side. One may proceed therefore by ignoring this term in a first approximation. After $g(\eta)$ has been calculated on this basis it would be possible to improve on the approximation by substituting $dg/d\eta$ on the right of equation (18) and calculate a second-order solution $g(\eta)$. Provided that the iterative procedure converged, any desired degree of accuracy could be attained in this manner.

At any stage of this iterative procedure the right-hand side of equation (18) is regarded as a known function of $\eta$, say $\psi(\eta)$. Then it is not difficult to show that the solution of the equation which makes $g(\eta)$ and $g'(\eta)$ tend to zero at large absolute values of $\eta$ is

$$g = e^{-k\eta} \int_{-\infty}^{\eta} e^{2ks}\,ds \int_{+\infty}^{s} \psi(t)\,e^{-kt}\,dt. \tag{19}$$

The first-order solution is obtained by setting

$$\psi = -F''^2/R_T\gamma. \tag{20}$$

As a very crude approximation, one may try to represent $F''(\eta)$ (the mean velocity gradient) by the following step function

$$\left.\begin{aligned} F'' &= -\frac{R_T^{\frac{1}{2}}}{2\pi^{\frac{1}{2}}} \quad \text{if} \quad \left(\frac{\pi}{R_T}\right)^{\frac{1}{2}} \leqslant \eta \leqslant +\left(\frac{\pi}{R_T}\right)^{\frac{1}{2}}, \\ &= 0 \text{ otherwise.} \end{aligned}\right\} \tag{21}$$

This value of $F''$ agrees with the theoretical result of Görtler at $\eta = 0$, where the velocity gradient is a maximum. The turbulent intensity calculated from this approximation is therefore likely to be an overestimate, but should at least provide qualitative information on the effects of the constants $\gamma$, $\Lambda$ and $R_T$ characterizing diffusion, dissipation and production of turbulent energy, respectively. When the integrations indicated in equation (19) are carried out, one finds that

$$\left.\begin{aligned} g &= \frac{e^{-k|\eta|}}{4\pi\Lambda} \sinh\left(\frac{\Lambda\pi}{\gamma R_T}\right)^{\frac{1}{2}} \quad \left[|\eta| \geqslant \left(\frac{\pi}{R_T}\right)^{\frac{1}{2}}\right], \\ &= \frac{1}{4\pi\Lambda}\left[1-\exp\left\{-\left(\frac{\pi\Lambda}{\gamma R_T}\right)^{\frac{1}{2}}\right\}\cosh\left[(\Lambda/\gamma)^{\frac{1}{2}}\eta\right]\right] \quad \left[|\eta| \leqslant \left(\frac{\pi}{R_T}\right)^{\frac{1}{2}}\right]. \end{aligned}\right\} \tag{22}$$

Thus the maximum value of $g$ is given by

$$g_{\max} = \frac{1-\exp\left[-(\pi\Lambda/\gamma R_T)^{\frac{1}{2}}\right]}{4\pi\Lambda}. \tag{23}$$

When the observed values $\gamma = \frac{1}{600}$, $\Lambda = \frac{1}{3}$, $R_T = 600$ are substituted into this formula one finds $g_{max} = 0{\cdot}15$, which is about 3 times the observed value, certainly a strong overestimate. As remarked before, the practical value of this result lies mainly in providing a fairly simple basis for estimating the probable effects of changing turbulence constants. In particular, it is to be noted that if the product $\gamma R_T$ is doubled, while $\Lambda$ stays constant, the maximum turbulent intensity, as given by equation (13), drops by 20 %.

A better estimate of $F''^2$ is provided by Görtler's first-order solution, which gives, in place of equation (21),

$$\psi = -\frac{1}{4\pi\gamma}\exp\left(-\tfrac{1}{2}R_T\eta^2\right). \tag{24}$$

The comparison of Görtler's solution with experimental values suggests that the Gaussian curve *underestimates* the velocity gradient, but not as much as equation (21) overestimates it. After the integrations are carried out one finds

$$\begin{aligned} g = \frac{\exp(\Lambda/2\gamma R_T)}{4(2\pi\gamma\Lambda R_T)^{\frac{1}{2}}}&\left[\cosh\left[(\Lambda/\gamma)^{\frac{1}{2}}\eta\right] + \tfrac{1}{2}\exp\{-(\Lambda/\gamma)^{\frac{1}{2}}\eta\}\,\phi\{(\tfrac{1}{2}R_T)^{\frac{1}{2}}\eta - (\Lambda/2\gamma R_T)^{\frac{1}{2}}\}\right. \\ &\left. - \tfrac{1}{2}\exp\{(\Lambda/\gamma)^{\frac{1}{2}}\eta\}\,\phi\{(\tfrac{1}{2}R_T)^{\frac{1}{2}}\eta + (\Lambda/2\gamma R_T)^{\frac{1}{2}}\}\right], \end{aligned} \tag{25}$$

with $\phi(t) = \dfrac{2}{\pi^{\frac{1}{2}}}\displaystyle\int_0^t \exp(-p^2)\,dp.$

The maximum value of $g$, at $\eta = 0$, becomes now

$$g_{\max} = \frac{\exp(\Lambda/2\gamma R_T)}{4(2\pi\gamma\Lambda R_T)^{\frac{1}{2}}}[1-\phi\{(\Lambda/2\gamma R_T)^{\frac{1}{2}}\}]. \tag{26}$$

When the estimates of the constants used above are substituted one finds $g_{\max} = 0{\cdot}115$, which is still more than twice the value observed by Liepmann & Laufer. Since $F''$ as used in this last calculation is an underestimate, the discrepancy between theory and observation must now be attributed to a neglect of the 'damping' term in equation (18), the term containing $dg/d\eta$. A little reflection shows that the turbulent intensity distribution of equation (25) in fact predicts some steep gradients $dg/d\eta$, which would act as 'negative' sources.

Again, the main point of this result is a prediction of the variation of $g_{\max}$ with varying turbulence constants. With $\gamma R_T$ doubled, equation (26) shows a 22 % drop in turbulent intensity, almost exactly as with the earlier approximation.

An interesting feature of the result (either equation (25) or (26)) is that the two constants $\gamma R_T$ only occur in combination, as a product. From the definitions of these constants (equations (6) and (17)) it is at once obvious that the product $\gamma R_T$ is the ratio of the exchange coefficient $D$ (diffusivity) of turbulent energy to the eddy kinematic viscosity $\nu_T$. Such a ratio is akin to the Prandtl number of the turbulence. Using the numerical values derived above from the experimental results of Liepmann & Laufer one finds a value of unity for the ratio $D/\nu_T$. Townsend (1956) also quotes some results of heat diffusion measurements showing a turbulent Prandtl number close to $1{\cdot}0$, which is also the basic assumption underlying 'Reynolds's analogy' in turbulent heat transfer.

The central idea of the experimental investigations described below was that by means of extraneous vortices the diffusive capacity of turbulence for turbulent energy could possibly be increased without affecting the other parameters. The theory now shows that the exchange coefficient *ratio* $D/\nu_T$ has to be increased very considerably for a significant effect on the peak turbulent intensity. The possibility of this seems remote when one considers that the turbulent Prandtl number moves within a narrow range in several different kinds of turbulent flow.

As a check on the qualitative correctness of the above theory the intensity profile predicted by equation (25) may now be compared with experiment. This has been done in figure 2. The difference in the location of the maxima is insignificant, being due to an arbitrary choice of zeros in the experiments. The curve labelled 'theory' has been obtained with the aid of the constants estimated before and no attempt was made to produce better agreement by adjusting the constants. A different choice of the value for the dissipation constant $\Lambda$ would be particularly powerful in producing better agreement. For example, setting $\Lambda = 0{\cdot}5$ in place of $0{\cdot}33$ yields a peak turbulent intensity of $0{\cdot}0873$ which is much closer to the measured values than $0{\cdot}115$, obtained with $\Lambda = 0{\cdot}33$. However, the qualitative correctness of the theory is obvious from the figure without adjustments and it is equally clear that a close fit with experiment may be obtained by an appropriate adjustment of the three constants $\gamma$, $R_T$ and $\Lambda$.

Direct numerical integration of equation (18), (without neglecting the $dg/d\eta$ term) may be expected to produce much better correspondence with observation than the above calculations. This would, however, have relatively little significance because the theory itself has been derived from an analysis of the observations. The point of the above calculations is that they yielded an explicit (if approximate) formula for assessing the influence of changing turbulence constants $\gamma$, $R_T$, $\Lambda$, on the peak intensity.

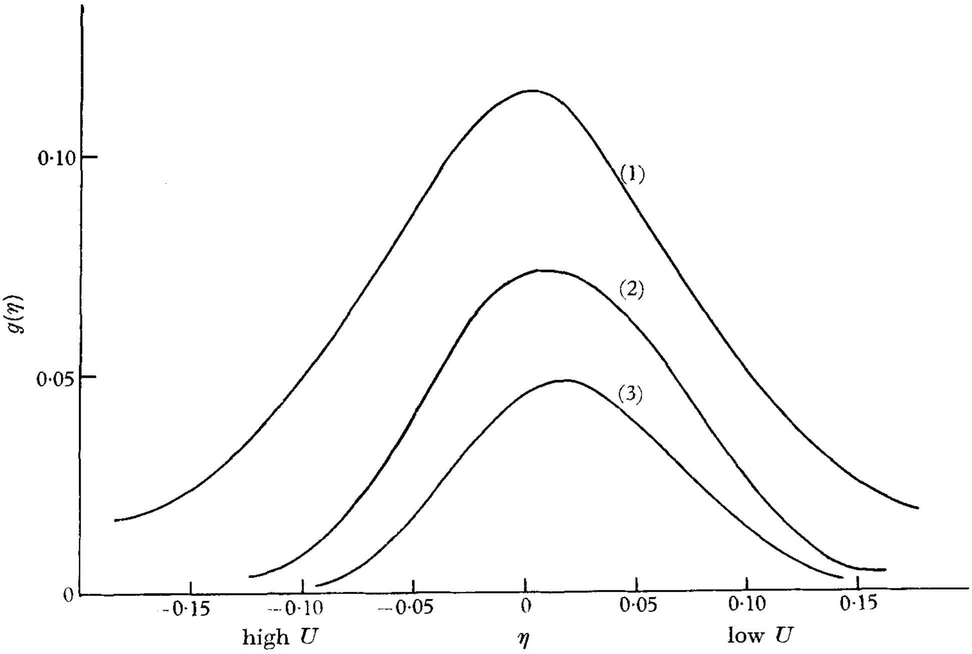


FIGURE 2. Turbulent intensity profile in the mixing layer: (1) theory (no adjustable constants), (2) present measurements, (3) Townsend.

## 9. The experimental arrangement

In order to obtain experimental data for this investigation a small open-jet wind tunnel has been constructed at Essex College. The details of the equipment have been described in a report of limited circulation and here it is only intended to summarize the more important features.

For the present investigations it was felt that some simple and cheap apparatus would be satisfactory. The properties of the free-mixing layer are known from the accurate work of Liepmann & Laufer (1947) and the main interest in the present case centres on how these properties could be modified. Therefore no great stress was laid on keeping the mean velocity of the merging jet exactly uniform (there are departures amounting to $\pm 5\%$ from the mean) nor was it attempted to produce a low turbulence level in the jet. (At nozzle exit the ratio R.M.S. velocity to mean velocity was about $1{\cdot}5\%$). However, the 2:1 reduction section of the nozzle was designed using perfect fluid theory to produce a steady pressure drop along the wall with the effect that the boundary-layer thickness at the exit was of the same low order of magnitude as in the experiments of Liepman & Laufer. The nominal size of the exit nozzle is $19\frac{3}{4}$ in. in the vertical by $7\frac{5}{8}$ in. in the horizontal. The measurements were carried out on one side, about 6 in. above the bottom of the duct at a jet velocity of approximately 70 ft./sec.

One important difference as against the measurements of Liepmann & Laufer was that mixing layers were allowed to develop on *both* sides of the plane jet. Both Reichardt (see Schlichting 1960) and Liepmann & Laufer (1947) state that it is important to prevent the formation of the second mixing layer by joining one nozzle wall to a flat plate. Presumably it is then possible to follow the development of one mixing layer for a larger distance axially. It is, however, conceivable that a somewhat different large-eddy pattern will be set up by this asymmetric arrangement than in a regular jet, although on the basis of the theoretical results in § 8 one would not expect this to exert a significant influence. If the mixing layer studies are to mean anything, they should be applicable to two-sided jets, possibly with some minor modifications.

As vortex generators equilateral triangles of sides 4 in. ('small') and 6 in. ('large') were used, set at small incidences against the stream in the manner of delta wings. They were supported on flat plates held parallel to the stream so as to avoid causing a flow disturbance.

The core of the instrumentation is a DISA constant-temperature hot-wire anemometer (described by Kidron 1960). This enables direct readings of mean velocity and root-mean-square turbulent velocity to be taken and an outlet may also be connected to an oscilloscope for qualitative investigations of the turbulence. Under the test conditions, the frequency response of the instrument was virtually perfect up to 10 kc/s, when, in a typical location, 93 % of the turbulent energy was contained in the frequency range below 1 kc/s. The noise level was quite negligible in comparison with the fluctuations produced by the turbulence.

## 10. The experimental results

### *Self-preservation*

According to the results of Liepmann & Laufer the mixing layer should be very nearly self-preserving beyond $R_x = 4\times10^5$, or, in the present case, beyond about 10 in. downstream of the nozzle exit. One way to check this is to show $(\overline{u^2}/U_1^2)^{\frac{1}{2}}$ against $U/U_1$ in a graph, as has been done in figure 3, for the three axial stations $x = 10$ in., 15in. and 20 in. The profiles are identical at $x = 15$ in. and 20 in. and there are only slight discrepancies at $x = 10$ in. Further downstream the two mixing layers begin influencing each other, but in the self-preserving range the test results should be comparable to those of Liepmann & Laufer. All subsequently reported readings were taken at $x = 15$ in. With vortex generators present, self-preservation cannot, of course, be expected to hold.

### *Mean velocities*

Figure 4 contains mean velocity profiles obtained with and without vortex generators. The profile without vortex generators is *identical* with that measured by Liepmann & Laufer. The other three cases are: $\odot$ too little extra mixing to affect the mean velocity gradient, the vortex generators only produce a lateral displacement of the mixing layer; the points △ showing lateral displacement plus a considerable reduction of the velocity gradient; ⊕ too much interference with the mixing layer leading to an irregular profile. The abscissa is cross-flow distance measured from an arbitrary origin.

*R.M.S. turbulent velocities (axial component u only)*

These were measured simultaneously with the mean velocities and are shown in figure 5. The mean-square turbulent velocity without vortex generators is

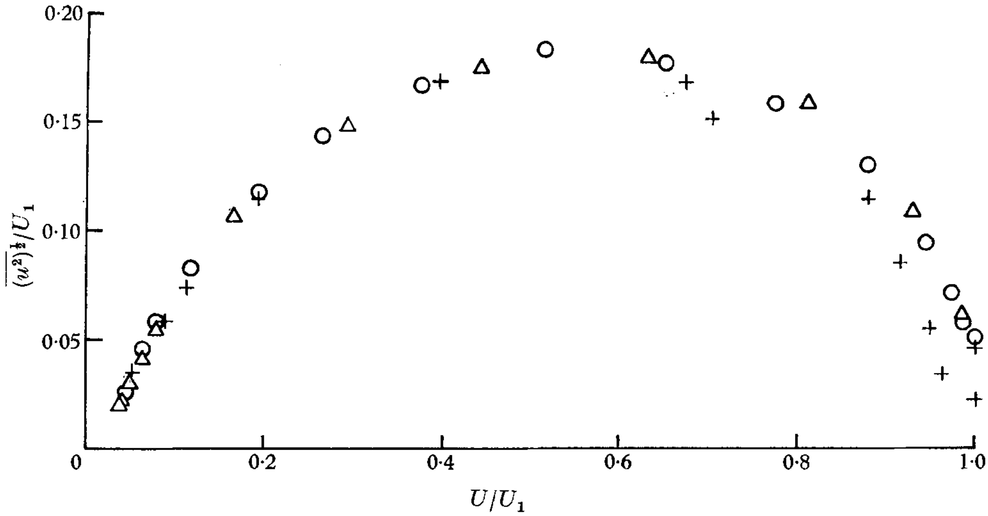


FIGURE 3. Turbulent velocities shown against mean velocities at three axial stations. ○, 20 in.; +, 10 in.; △, 15 in.

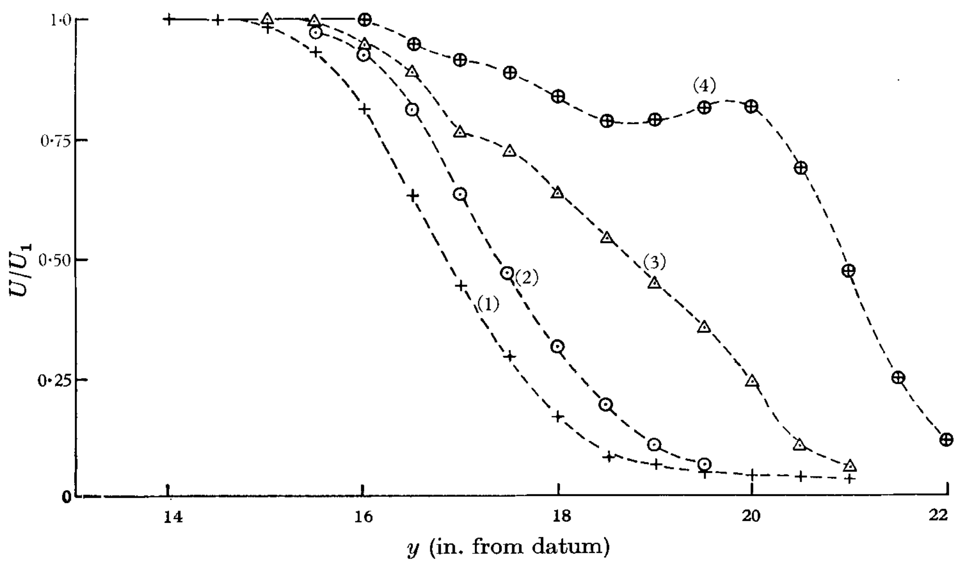


FIGURE 4. Mean velocities with and without vortex generators: (1) no vortex generators, (2) small vortex generators, low incidence, (3) large vortex generators, low incidence, (4) large vortex generators, high incidence.

also shown in figure 2 (labelled 'present measurements'), where it may be seen that this profile is *not* identical with that measured by Liepmann & Laufer. The values in figure 2 have been calculated on the assumption that $\overline{q^2} = 2{\cdot}25\overline{u^2}$ (equation (13) above). The discrepancy of the two experimental curves is quite puzzling particularly in view of the agreement of mean velocity profiles and of the self-preserving range.

The vortex generators are seen to produce a slight increase in the peak turbulence levels and a broadening of the turbulent region.

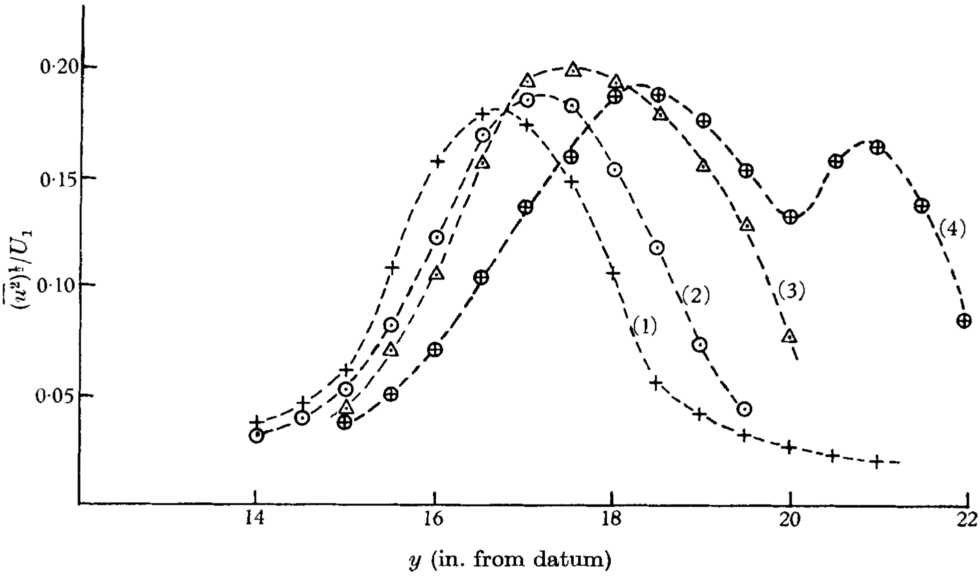


FIGURE 5. Turbulent velocities with and without vortex generators. Curves numbered as in figure 4.

## *Structure of turbulence*

An effort was made to explore qualitatively any changes in the turbulent structure that may have been caused by the vortex generators by means of oscilloscope trace photographs. No obvious differences could be detected in this manner. The intermittency of the turbulence was equally evident in the outer layers with or without vortex generators as was the existence of a broad range of Fourier components. Using the high pass filter on the instrument an integrated spectrum curve could be obtained. Figure 6 shows two such curves obtained with and without vortex generators, at points having the same mean flow velocity.

It is planned to conduct more detailed surveys of the turbulence when further instrumentation is available. Meanwhile, the above evidence suggests that the only appreciable effects of appropriate external vortices acting on a mixing layer are (1) a reduction of the mean velocity gradient, (2) a broadening of the turbulent region, and (3) a slight increase in peak turbulent intensity.

## 11. Discussion of the results

The one puzzling result of this investigation is that the peak root-mean-square turbulent velocity was found to be about 20 % higher than the values of Liepmann & Laufer. It is quite unlikely that the differences in the arrangement, notably the 'two-sidedness' of the jet, could be responsible for the difference. First, it is difficult to believe that any such influences would act on the turbulent intensity profile only, not on mean velocities. Secondly, the root-mean-square turbulent velocity changed only by about 10 % or less under the effect of an external disturbance as violent as a relatively large vortex generator. While the matter remains *sub judice*, it appears at present that the difference in measured turbulent intensities is instrumental in origin.

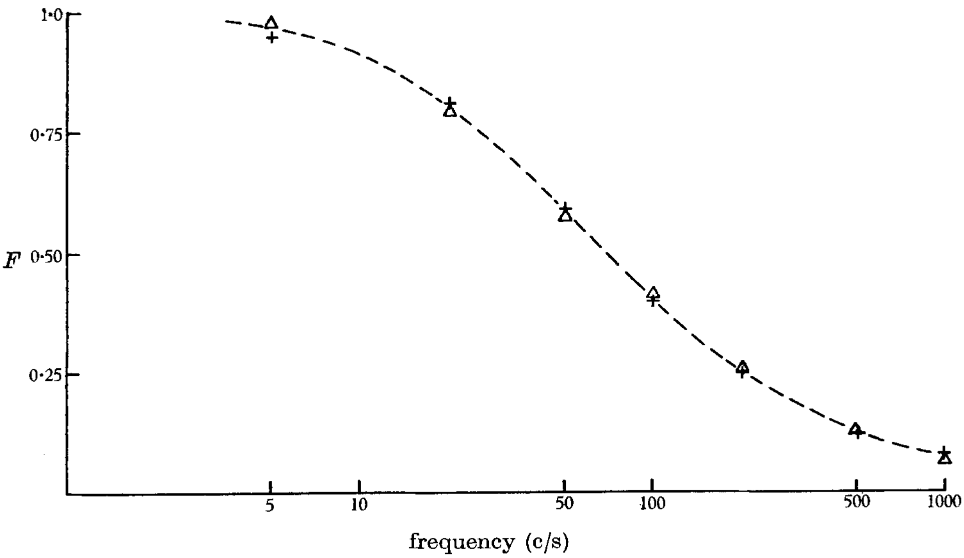


FIGURE 6. Integrated turbulence spectrum with and without vortex generators. Vortex generators: + without, Δ with.

As to the influence of the vortex generators on the mixing layer, the two major effects observed were a broadening of the turbulent region and a reduction in mean velocity gradient. It is reasonable to assume that the turbulent shear stress remained constant or at least did not decrease, because the slowing down of the jet had to take place in an equal or a shorter axial distance than without the vortex generators. It follows then that the eddy viscosity had to increase and the proportionate change appears to be very much the same as for the eddy diffusivity, the increase of which may be held responsible for the broadening of the turbulent region. These are plausible hypotheses, subject to confirmation in the course of more detailed surveys. At the very least it is unlikely that the introduction of extraneous vortices has increased the ratio of eddy diffusivity to eddy viscosity.

On this basis the theory shows that the peak turbulent intensity should not decrease and this is well borne out by experiment. In fact there is a moderate increase but the exact cause of this cannot be pin-pointed on the basis of the observations reported herein. It should be noted that the theoretical analysis based on self-preservation can only offer guidance as to what to expect in a less regular arrangement.

It is of some interest to point out the correspondence of these results with the observations of Corcos (1959) on corrugated nozzles. Comparing the turbulence of a circular and a corrugated nozzle (such as those found on aircraft) Corcos finds that the peak turbulent intensities near the 'exposed' tips of the corrugations are at least as high as those found in the mixing layer of a circular nozzle. The important difference is that these high turbulent intensities only occur over quite small regions, for they are not present in the space between two corrugations.

The above investigation was supported by a grant from the National Research Council of Canada, which is gratefully acknowledged.

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# CORRIGENDUM

'The characteristics of the turbulence in the mixing region of a round jet', by P. O. A. L. Davies, M. J. Fisher and M. J. Barratt, *J. Fluid Mech.* **15**, 1963, pp. 337–67.

The ordinate scale for the inverse of the shear on figure 12 is incorrect as drawn, and should be multiplied by the factor $\frac{2}{3}$ so that the times read 33, 67, 100 $\mu$sec instead of 50, 100, 150, etc. There is a corresponding alteration to equation (4.2), which should now be

$$L'_\tau \simeq 4{\cdot}5/(\partial U/\partial x_2). \tag{4.2}$$

Equation (4.6) should read as

$$(\overline{v_1^2})^{\frac{1}{2}}\, L'_\tau/L_{x_1} = 0{\cdot}9 \tag{4.6}$$

and correspondingly the ordinate scale on figure 19 should be multiplied by $\frac{3}{2}$ to read 0·3, 0·6, 0·9, etc.